Penguin amplitudes in $B^+ \to \pi^+ K^{*0}, K^+ \bar{K}^{*0}$ decays

Piotr Żenczykowski
Division of Theoretical Physics,
the Henryk Niewodniczański Institute of Nuclear Physics,
Polish Academy of Sciences,
Radzikowskiego 152, 31-342 Kraków, Poland

October 28, 2010

Abstract

The question of the relative size of two independent penguin amplitudes is studied using the data on the $B^+ \to \pi^+ K^{*0}$, and $B^+ \to K^+ \bar{K}^{*0}$ decays. Our discussion involves a Regge-phenomenology-based estimate of $SU(3)$ breaking in the final quark-pair-creating hadronization process. The results are in agreement with earlier estimates of the relative size of the two penguins obtained from $B^+ \to \pi^+ K^0, K^+ \bar{K}^0$.

PACS: 13.25.Hw; 12.15.Ji; 12.40.Nn; 14.40.Nd

1 Introduction

Rare charmless nonleptonic B meson decays give us a lot of information on the weak interactions of quarks. Yet, since quarks are forever confined, this information is necessarily blended with strong interaction effects. A proper and reliable understanding of the latter is crucial for the extraction of New Physics effects (if any). At present, however, the only way to achieve such an understanding is through the parametrization of strong interaction effects and the subsequent extraction of relevant parameters from the experimental data.

For example, it is known that the effective tree ($\tilde{T}$) and colour-suppressed ($\tilde{C}$) amplitudes involve contributions from some of the penguin amplitudes as well (see eg. [1, 2, 3]). Disentangling this penguin contribution from the 'true' C and T amplitudes would constitute part of the proper understanding of 'standard physics', both at the quark- and the hadron-level, and a check on theoretical predictions at the quark level. Unfortunately, achieving this goal is not possible on the basis of $B \to \pi\pi$ data alone, which permit an extraction of $\tilde{C}/\tilde{T}$, but not C/T. Indeed, estimating the latter would require knowledge of the size of one of the two independent penguin amplitudes [2]. Knowing the branching ratio of a single purely-penguin process is not sufficient here. If the two penguin contributions are to be separated properly, additional data and some related assumptions are needed.

The best place to analyse the size of penguin contributions should be in those processes in which only penguin amplitudes are present. For this reason, ref. [4] considered $B^+ \to \pi^+ K^0$ decays in conjunction with $B^+ \to K^+ \bar{K}^0$. In those two processes, amplitudes C and T are absent and only penguin amplitudes contribute. As shown in [4], under reasonable assumptions concerning SU(3) symmetry breaking, it is then possible to extract from the experimental data the relative size and phase of the two independent penguin amplitudes, and therefore estimate the effect of the penguin-induced corrections.

The present paper is concerned with the extraction of the ratio of two similar penguin amplitudes from the related, but independent, decays $B^+ \to \pi^+ K^{*0}$ and $B^+ \to K^+ \bar{K}^{*0}$. Our results are fully consistent with the findings of ref. [4].

2 Penguin amplitudes

For future comparison, we first briefly recall the notation used in [4] for the description of penguins in $B \to PP$ decays, and then introduce an analogous notation for $B \to PV$ processes.

2.1 $B \to PP$

Ref. [4] was concerned with the extraction of independent penguin amplitudes relevant for $B \to PP$ decays. In that case, when the elements of the CKM matrix

were factored out, the three independent strong penguin amplitudes corresponding to internal k-quark loops ($k = u, c, t$) were denoted in [4], [5] by $\mathcal{P}_k$. Because of the unitarity of the CKM matrix only two combinations of these amplitudes can be extracted from experimental data. For $b \to d$ transitions we choose the following two combinations ($q = u, c$) [1]:

$$P_q \equiv -\lambda_c^{(d)} \boldsymbol{\mathcal{P}}_{tq} = A\lambda^3 \boldsymbol{\mathcal{P}}_{tq}, \tag{1}$$

where

$$\boldsymbol{\mathcal{P}}_{tq} = \boldsymbol{\mathcal{P}}_t - \boldsymbol{\mathcal{P}}_q, \tag{2}$$

is the difference of two independent penguin amplitudes, and $\lambda_q^{(k)}$ is given in terms of the CKM matrix V:

$$\lambda_q^{(k)} = V_{qk} V_{qb}^*, \tag{3}$$

with A and $\lambda = 0.225$ being the Wolfenstein parameters (we do not need the actual value of A in our calculations).

If one assumes that the difference between the $b \to d$ and $b \to s$ penguin amplitudes arises solely from different CKM factors, the $b \to s$ transitions may be expressed in terms of the same strong penguin amplitudes $\boldsymbol{\mathcal{P}}_{tq}$ as the $b \to d$ processes. This assumption ignores any possible dependence on the spectator quark and on the flavour of the additional quark-antiquark pair which appears in the final state and has to be produced via strong interactions. While the spectator-independence of the amplitude seems a fairly reasonable assumption, the production of the additional quark-antiquark pair may be strongly flavour-dependent, as discussed at length in [4]. We will come to this point later. For the $b \to s$ processes, the corresponding products of strong penguin amplitudes and CKM factors will be denoted with primed letters, i.e.:

$$P'_q \equiv -\lambda_c^{(s)} \boldsymbol{\mathcal{P}}_{tq}, \tag{4}$$

so as to distinguish them from (unprimed) amplitudes P_q used for $b \to d$ decays.

We are interested in the relative size of the two independent combinations of penguin amplitudes, i.e. in the ratio:

$$\boldsymbol{\mathcal{P}}_{tu} / \boldsymbol{\mathcal{P}}_{tc}. \tag{5}$$

In order to discuss this ratio it is convenient to introduce [4]:

$$z e^{i\zeta} \equiv R_b \frac{P_u}{P_c} = R_b \frac{P'_u}{P'_c} = R_b \frac{\boldsymbol{\mathcal{P}}_{tu}}{\boldsymbol{\mathcal{P}}_{tc}}, \tag{6}$$

where [6]

$$R_b = \sqrt{\bar{\rho}^2 + \bar{\eta}^2} = -\frac{\lambda_u^{(d)}}{\lambda_c^{(d)}} e^{-i\gamma} \approx 0.38. \tag{7}$$

In our calculations we use the central value of the SM prediction $\gamma = 69.6^o \pm 3.1^o$, which compares very well with the value of $69.8^o \pm 3.0^o$ fitted in [6].

[1]In order to make our notation more transparent, all amplitudes denoted in [4] and [5] by calligraphic letters are written here in bold.

2.2 $B \to PV$

For the $B \to PV$ decays there are two possible cases: the spectator quark (i.e. the quark not involved in weak interactions) ends up either in a pseudoscalar (P) or in a vector (V) meson. In order to distinguish between these two cases, we introduce the subscript P or V in all relevant amplitudes (hence the corresponding amplitudes are distinguished from the $B \to PP$ amplitudes for which we do not use a subscript) [5]. For example, P_P (P_V) denotes full penguin amplitudes relevant in $B \to PV$ decays with spectator quark ending in a P (V) meson.

For the decays with the spectator quark emerging in the final pseudoscalar meson, and with the CKM matrix elements factored out, we have again three independent strong penguin amplitudes $\boldsymbol{\mathcal{P}}_{P,k}$ corresponding to k-quark running in an internal loop. As in the case of $B \to PP$ decays, we use the unitarity of the CKM matrix to describe the relevant total penguin amplitudes in terms of two combinations ($q = u, c$) analogous to $\boldsymbol{\mathcal{P}}_{tq}$ of Eq. (2):

$$\boldsymbol{\mathcal{P}}_{P,tq} = \boldsymbol{\mathcal{P}}_{P,t} - \boldsymbol{\mathcal{P}}_{P,q}. \tag{8}$$

The full penguin amplitudes with spectator quark ending in a pseudoscalar meson are then

$$\begin{aligned} P_P &= -\lambda_u^{(d)} \boldsymbol{\mathcal{P}}_{P,tu} - \lambda_c^{(d)} \boldsymbol{\mathcal{P}}_{P,tc}, && (9) \\ P'_P &= -\lambda_u^{(s)} \boldsymbol{\mathcal{P}}_{P,tu} - \lambda_c^{(s)} \boldsymbol{\mathcal{P}}_{P,tc}, && (10) \end{aligned}$$

for $b \to d$ and $b \to s$ transitions respectively. Again, the above formulae ignore any other possible dependence on the spectator quark and any dependence on the flavour of the quark-antiquark pair that has to be additionally produced via strong interactions.

In order to discuss the ratio of penguin amplitudes $\boldsymbol{\mathcal{P}}_{P,tu}/\boldsymbol{\mathcal{P}}_{P,tc}$ relevant in $B \to PV$ decays, and to compare it later with the $B \to PP$ case, it is convenient to define the counterpart of Eq. (6):

$$z_P e^{i\zeta_P} \equiv R_b \frac{P_{P,u}}{P_{P,c}} = R_b \frac{P'_{P,u}}{P'_{P,c}} = R_b \frac{\boldsymbol{\mathcal{P}}_{P,tu}}{\boldsymbol{\mathcal{P}}_{P,tc}}, \tag{11}$$

where $P_{P,q}$ and $P'_{P,q}$ are defined analogously to Eqs (1,4):

$$\begin{aligned} P_{P,q} &\equiv -\lambda_c^{(d)} \boldsymbol{\mathcal{P}}_{P,tq}, && (12) \\ P'_{P,q} &\equiv -\lambda_c^{(s)} \boldsymbol{\mathcal{P}}_{P,tq}. && (13) \end{aligned}$$

3 Decays $B^+ \to \pi^+ K^{*0}, K^+ \bar{K}^{*0}$

If there is no dependence on the flavour of the additional quark-antiquark pair produced via strong final state interactions, the amplitudes for the $B^+ \to \pi^+ K^{*0}$

and $B^+ \to K^+ \bar{K}^{*0}$ decays are:

$$A(B^+ \to \pi^+ K^{*0}) = P'_P, \tag{14}$$
$$A(B^+ \to K^+ \bar{K}^{*0}) = P_P. \tag{15}$$

Using Eqs (9,10) and the relation

$$P'_{P,c} = \frac{\lambda_c^{(s)}}{\lambda_c^{(d)}} P_{P,c} = -\frac{1}{\sqrt{\epsilon}} P_{P,c}, \tag{16}$$

where

$$\epsilon = \frac{\lambda^2}{1-\lambda^2} \approx 0.053, \tag{17}$$

one can reexpress the above amplitudes in terms of $P_{P,c}$, z_P and ζ_P as:

$$A(B^+ \to \pi^+ K^{*0}) = -\frac{1}{\sqrt{\epsilon}} P_{P,c}(1 + \epsilon z_P e^{i(\zeta_P+\gamma)}), \tag{18}$$
$$A(B^+ \to K^+ \bar{K}^{*0}) = P_{P,c}(1 - z_P e^{i(\zeta_P+\gamma)}). \tag{19}$$

The above formulae are completely analogous to those relevant for $B^+ \to \pi^+ K^0$ and $B^+ \to K^+ \bar{K}^0$ decays [4] (with the replacements $P_{P,c} \to P_c$, $z_P \to z$, and $\zeta_P \to \zeta$).

In reality, the decays $B^+ \to \pi^+ K^{*0}$ and $B^+ \to K^+ \bar{K}^{*0}$ may differ in an important way, not taken into account in the above formulae. Indeed, the newly created quark-antiquark pair is produced (long after the weak decay) by long-range strong interactions (see Fig. 1). Now, it is known that in such strong-interaction processes - in which newly produced quarks q and $\bar{q}$ end up in different hadrons - the production of the $s\bar{s}$ pair is strongly suppressed at high energies when compared to that of the light $q\bar{q}$ pair. In order to take this effect into account, we modify formula (19) by introducing a corresponding suppression factor κ:

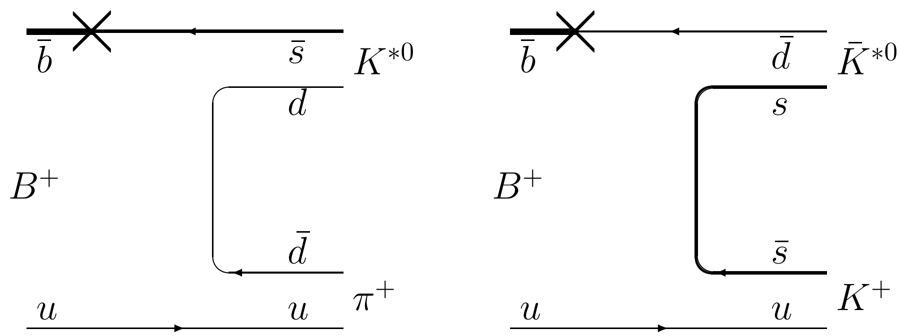
$$A(B^+ \to K^+ \bar{K}^{*0}) = \kappa P_{P,c}(1 - z_P e^{i(\zeta_P+\gamma)}). \tag{20}$$

A reliable estimate of this suppression factor may be obtained using Regge phenomenology as discussed in much detail in ref. [4], where it was shown that

$$\kappa = (m_B^2/s_0)^{\alpha_0(K^*)-\alpha_0(\rho)}, \tag{21}$$

with $s_0 \approx 1\ GeV^{-2}$, and $\alpha_0(M)$ being the intercept of Regge trajectory corresponding to meson M. Since the values of the intercepts are extracted directly from high energy scattering experiments, their values take into account *all* final state interactions. For $m_B^2 = 27.9\ GeV^2$ one finds then that $\kappa \approx 0.5 - 0.6$ [4, 7, 8]. In the following we take $\kappa = 0.55$.

Figure 1: B^+ decays to $\pi^+ K^{*0}$ and $K^+ \bar{K}^{*0}$ final states. Penguin $\bar{b} \to \bar{s}$ and $\bar{b} \to \bar{d}$ transitions are denoted with crosses.



3.1 Extraction of penguins' relative sizes and phases

The CP-averaged branching ratios for the $B^+ \to \pi^+ K^{*0}, K^+ \bar{K}^{*0}$ decays are given by

$$\begin{aligned} \langle \mathcal{B}(B^+ \to \pi^+ K^{*0}) \rangle_{CP} &\approx \frac{1}{\epsilon} |P_{P,c}|^2 \, (1 + 2\, \epsilon \, z_P \, \cos\zeta_P \cos\gamma), \\ \langle \mathcal{B}(B^+ \to K^+ \bar{K}^{*0}) \rangle_{CP} &= \kappa^2 |P_{P,c}|^2 \, (1 + z_P^2 - 2 z_P \cos\zeta_P \cos\gamma), \end{aligned} \tag{22}$$

where we neglected the terms of order $(\epsilon z_P)^2$. The latter assumption is well justified since $\epsilon \approx 1/20$ and

$$z_P = 0.38 \left| \frac{\boldsymbol{\mathcal{P}}_{P,t} - \boldsymbol{\mathcal{P}}_{P,u}}{\boldsymbol{\mathcal{P}}_{P,t} - \boldsymbol{\mathcal{P}}_{P,c}} \right| \tag{23}$$

is expected to be of order 1 (or smaller if the top quark penguin $\boldsymbol{\mathcal{P}}_{P,t}$ dominates).

We now take the ratio of the two branching fractions to find that

$$R^{KK^*}_{\pi K^*} \equiv \frac{\langle \mathcal{B}(B^+ \to K^+ \bar{K}^{*0}) \rangle_{CP}}{\langle \mathcal{B}(B^+ \to \pi^+ K^{*0}) \rangle_{CP}} = \epsilon \, \kappa^2 \, \frac{1 + z_P^2 - 2 z_P \cos\zeta_P \cos\gamma}{1 + 2\, \epsilon \, z_P \, \cos\zeta_P \cos\gamma} \tag{24}$$

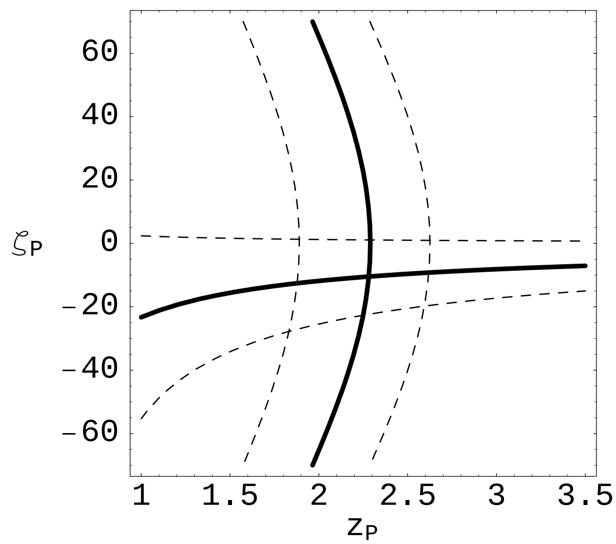
(thus, we do not need the value of $|P_{P,c}|$).

The averages of experimental branching ratios for $B^+ \to \pi^+ K^{*0}$ and $B^+ \to K^+ \bar{K}^{*0}$ decays are (in units of 10^{-6}) [9]:

$$\begin{aligned} \langle \mathcal{B}(B^+ \to \pi^+ K^{*0}) \rangle_{CP} &= 9.9^{+0.8}_{-0.9}, \\ \langle \mathcal{B}(B^+ \to K^+ \bar{K}^{*0}) \rangle_{CP} &= 0.68 \pm 0.19. \end{aligned} \tag{25}$$

corresponding to $R^{KK^*}_{\pi K^*} = 0.069 \pm 0.020$.

Figure 2: Contour plot showing the experimentally allowed region of (z_P, ζ_P) plane. Approximately vertical (horizontal) lines correspond to branching ratio (asymmetry) constraints. Solid lines represent constraints for central experimental values, dashed lines - for one standard deviation.



For the asymmetries A_{CP} we find:

$$A_{CP}(B^+ \to \pi^+ K^{*0}) = \frac{2\,\epsilon z_P\, \sin\zeta_P \sin\gamma}{1 + 2\,\epsilon z_P \cos\zeta_P \cos\gamma}, \tag{26}$$

$$A_{CP}(B^+ \to K^+ \bar{K}^{*0}) = -\frac{2\, z_P \sin\zeta_P \sin\gamma}{1 + z_P^2 - 2\, z_P \cos\zeta_P \cos\gamma}, \tag{27}$$

while the experimental value is known for the first process only, with the average [9]:

$$A_{CP}(B^+ \to \pi^+ K^{*0}) = -0.038 \pm 0.042. \tag{28}$$

Equations (24,25,26,28) provide two conditions on z_P and ζ_P. Their solution yields:

$$\begin{aligned} z_P &= 2.3 \pm 0.3, \\ \zeta_P &= -11^o \pm 12^o, \end{aligned} \tag{29}$$

or

$$\begin{aligned} z_P &= 1.45^{+0.30}_{-0.40}, \\ \zeta_P &= -165^o \pm 15^o. \end{aligned} \tag{30}$$

The second solution corresponds to $\boldsymbol{\mathcal{P}}_{P,tu}$ and $\boldsymbol{\mathcal{P}}_{P,tc}$ with nearly opposite phases. This is highly unlikely since the masses of u and c quarks are small when compared to the mass of t quark, and - consequently - the two penguins should have similar strong phases. Therefore, this solution may be discarded. For the first solution, the relevant contour plot in the (z_P, ζ_P) plane is shown in Fig. 2.

The values of z_P, ζ_P extracted from experimental data on $B \to \pi K^*, KK^*$ decays (Eq.(29)) are very similar to the values of

$$\begin{aligned} z &= 1.8 - 2.3, \\ \zeta &= -15^o \text{ to } 0^o, \end{aligned} \tag{31}$$

obtained in [4] in a similar analysis of the $B \to \pi K, KK$ decays.

This agreement corroborates the idea that there is no essential difference between the short-range penguin transitions in $B \to PP$ and $B \to PV$ decays. The only important difference between the pure penguin-driven processes in $B \to PP$ and $B \to PV$ processes arises from final-state long-range interactions. In particular, the relative size of the relevant branching ratios indicates that one has to include the effect of $SU(3)$ breaking ($\kappa \neq 1$) in the final $q\bar{q}$-creation process.

4 Discussion and conclusions

It is straightforward to derive the following formula for the $A_{CP}(B^+ \to K^+\bar{K}^{*0})$ asymmetry:

$$A_{CP}(B^+ \to K^+\bar{K}^{*0}) = -\kappa^2 \frac{\langle \mathcal{B}(B^+ \to \pi^+ K^{*0})\rangle_{CP}}{\langle \mathcal{B}(B^+ \to K^+\bar{K}^{*0})\rangle_{CP}} A_{CP}(B^+ \to \pi^+ K^{*0}). \tag{32}$$

For $\kappa = 0.55$ one has

$$A_{CP}(B^+ \to K^+\bar{K}^{*0}) \approx +0.17 \pm 0.19, \tag{33}$$

with the dominant contribution to the error coming from $A_{CP}(B^+ \to \pi^+ K^{*0})$. The asymmetry $A_{CP}(B^+ \to K^+\bar{K}^{*0})$ is expected to be some four times larger than $A_{CP}(B^+ \to \pi^+ K^{*0})$. This is because it is not proportional to the suppression factor of ϵ: the two contributing penguin amplitudes are of comparable sizes. Consequently, it would be interesting to have $A_{CP}(B^+ \to K^+\bar{K}^{*0})$ measured as it is particularly sensitive to the size of the relative strong phase of the two penguin contributions.

We have found that the ratios of the two penguin amplitudes are similar in the $B^+ \to \pi^+ K^0$ ($K^+\bar{K}^0$) and $B^+ \to \pi^+ K^{*0}$ ($K^+\bar{K}^{*0}$) sectors, i.e. that they are independent of the final state, and that the SU(3) breaking factor κ is universal. Consequently, it is natural to expect that the same will be true for the $B^+ \to \rho^+ K^0$ ($K^{*+}\bar{K}^0$) sector, where the spectator quark ends up in a vector meson (i.e. that $z_P \approx z \approx z_V$, and $\zeta_P \approx \zeta \approx \zeta_V$).

This leads us to expect that the branching fraction of the $B \to K^{*+}\bar{K}^0$ decay should be approximately equal to

$$\langle \mathcal{B}(B^+ \to K^{*+}\bar{K}^0)\rangle_{CP} = \langle \mathcal{B}(B \to \rho^+ K^0)\rangle_{CP} \frac{\langle \mathcal{B}(B^+ \to K^+\bar{K}^{*0})\rangle_{CP}}{\langle \mathcal{B}(B^+ \to \pi^+ K^{*0})\rangle_{CP}} \approx 0.55 \pm 0.2. \tag{34}$$

An analogon of Eq. (32) may also be written.

It is hoped that future experimental work on $B^+ \to K^{*+}\bar{K}^0$, $B^+ \to K^+\bar{K}^{*0}$, $B^+ \to \pi^+ K^{*0}$, and $B \to \rho^+ K^0$ processes will further corroborate the ideas of this paper.

5 Acknowledgements

I would like to thank Leonard Leśniak for reading the manuscript prior to publication. This work has been partially supported by the Polish Ministry of Science and Higher Education research project No N N202 248135.

References

[1] M. Gronau, O. F. Hernandez, D. London, and J. L. Rosner, Phys. Rev. **D50**, 4529 (1994).

[2] A. J. Buras, R. Fleischer, S. Recksiegel, and F. Schwab, Phys. Rev. Lett. **92**, 101804 (2004); Nucl. Phys. **B697**, 133 (2004).

[3] C.-W.Chiang, M. Gronau, J. L. Rosner, and D. A. Suprun, Phys. Rev. **D70**, 034020 (2004).

[4] P. Żenczykowski, Acta Phys. Pol. B41 (2010) 79.

[5] M. Sowa, P. Żenczykowski, Phys. Rev. **D71**, 114017 (2005).

[6] UT$_{fit}$ Collaboration, http://www.utfit.org/UTfit/Results

[7] A. C. Irving and R. P. Worden, Phys. Rep. **34**, 117 (1977).

[8] A. D. Martin, C. Michael, and R. J. N. Phillips, Nucl. Phys. **43B**, 13 (1972).

[9] Heavy Flavor Averaging Group (HFAG),
http://www.slac.stanford.edu/xorg/hfag/rare/ichep10/charmless/index.html
http://www.slac.stanford.edu/xorg/hfag/rare/ichep10/acp/index.html